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Analysing spectra on production of light clusters in heavy-ion collisions and reflections on the average phase space density

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Abstract. In the recent past the NA44 Collaboration reported measurements on production of deuteron spectra in SS , SPb and PbPb collisions at the CERN-SPS. This has opened up an avenue, within the confines of the coalescence model, for calculating the source radii of protons, antiprotons and the average phase space densities (APSD) which are being projected, of late, as a reliable observable for studying the freeze-out behaviour of high-energy nuclear collisions. Instead of the thermal or thermodynamical model, we have applied in the present work a set of two models with a new parametrisation which yielded in the past a fair description of some observations on the APSD nature for pion production in nuclear collision. The combination provides here a satisfactory analysis of the p_{T} spectra for both proton and deuteron production and allows us to obtain the values of the hypothesized radii of the assumed spheres of the expansion dynamics and also of the average phase space densities for protons. The values thus arrived at are also quite consistent with the RQMD predictions in the standard literature.

PACS. 13.60.Hb Total and inclusive cross-section (including deep-inelastic processes) – 25.75.-q Relativistic heavy-ion collisions

1 Introduction

Relativistic heavy-ion collisions offer us an enormous opportunity to study the behaviour of particle production at very high densities and temperatures comparable to those of the early universe when it was supposedly in the state of plasma comprising free quarks and gluons. The particles are produced when the dense, hot plasma cools down and the "freeze-out" occurs. And the process of freeze-out gives rise to various hadrons and hadronic clusters. The lightest known cluster is that of deuteron. The coalescence of nucleons into deuterons is sensitive to both their spatial and momentum correlations. In this paper we would concentrate on some aspects of the source sizes of the proton and deuteron production in SS, SPb and PbPb collisions; and combine them with single-particle spectra to derive average phase space densities.

Let us now make a few points somewhat categorically, albeit, first in a negativistic and preemptive manner. Primarily, our intention here is not to produce any physically motivated picture of cluster production or any absolutely new dynamics of heavy-ion collisions. Secondly, we are not interested here in suggesting any mechanisms for refining or modifying the coalescence picture as such. But we would certainly utilize this coalescence picture for an altogether different objective. This is to test the efficacy of a new parametrization, called De-Bhattacharyya parametrization (DBP), in explaining the features of measurements related to the observables on deuteron production. This has been dwelt upon in some detail in a subsequent section. The physics behind the veil and validity of this parametrization may lead in the future, we hope, to radically new insights into cluster production in AB/AA collisions, but not immediately. This apart, we would also try to provide a framework for analysing and obtaining the results reported so far by RQMD and also by some measurements done in a limited way. Besides, we would also like to check the average phase space density values for protons which are deduced here with an approach parallel to what was earlier adopted for pions and was reported by us [1,2] in previous papers.

This work is organised as follows. In sect. 2 we introduce the coalescence formalism. In the subsequent section (sect. 3) a brief sketch on the physics of the average phase space density of the proton is dealt with. In the next section (sect. 4) we present the outline of the combination of

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the phenomenological models and of the new parametrization that are to be applied here. In sect. 5 we provide some important clues to the physical understanding of the nature of some parameters used in our work here and to their hidden implications. The sixth section contains the results of our work done here and their diagrammatic comparison with RQMD projections. The following sect. 7 gives the summary and final discussion.

2 General theoretical framework: An outline

It is well known that the studies on deuteron production are commonly grounded on the tacit acceptance of the coalescence picture. According to this view, deuteron production with a certain velocity is proportional to the number of protons and neutrons that have similar velocities; and the coalescence factor is contingent upon the distribution of nucleons. This property guides us to determine the source size of the nucleon from the ratio defined by the form given below [3,4]

$$B_2(p) = \frac{E_d \frac{\mathrm{d}^3 N_d}{\mathrm{d}^3 P}}{E_p \frac{\mathrm{d}^3 N_p}{\mathrm{d}^3 p} E_n \frac{\mathrm{d}^3 N_n}{\mathrm{d}^3 p}}\,, \tag{1}$$

where the deuteron momentum P is twice the proton momentum p. Measurement of neutrons is normally avoided. In bypassing it, two modestly valid assumptions on the shape of the neutron spectrum and another on the magnitude of the n/p ratio are made. They are: i) the spectrum shape of the neutron is identical to that of the proton; ii) the magnitude of the n/p ratio is assumed here to be ~ 1.06 [5] in order to give the present work a basis for direct comparison with RQMD.

It is seen from the fig. 2 of Murray and Holzer's work [6] that there is no appreciable difference in the magnitudes of the radius values between a Gaussian and a non-Gaussian (Hulthen) form. So with a view to avoiding unnecessary complications, we have chosen here the plain Gaussian source. And on the acceptance of this Gaussian wave function, the nature of the relationship between the coalescence factor(B_2) and the source radius(R_G) is given by [6],

$$\left(R_G^2 + \frac{\delta^2}{2}\right)^{3/2} = \frac{3\pi^{3/2}(c\hbar)^3}{2m_p B_2}\,, \tag{2}$$

where m_p is the proton mass, and $\delta = 2.8$ fm provides the size of the deuteron.

3 Average phase space density: A definition

A particle phase space density is defined as [6]

$$f(\mathbf{p}, \mathbf{x}) = (2\pi\hbar c)^3 \frac{\mathrm{d}^6 N}{\mathrm{d}^3 p \mathrm{d}^3 x}\,. \tag{3}$$

For a system in chemical equilibrium at a temperature T and chemical potential μ,

$$f(E) = \frac{1}{\exp{(E-\mu)} \pm 1}\,, \tag{4}$$

where E is the energy and ± 1 selects bosons or fermions. For a dilute system, *i.e.*, $f \ll 1$ the above equation gives

$$f_d \approx \exp - \frac{(E_d - \mu_p - \mu_n)}{T}\,. \tag{5}$$

Since $E_d = E_n + E_p$, the above equation assumes the following form:

$$f_d(\mathbf{P}, \mathbf{x}) = f_p(\mathbf{p}, \mathbf{x}) f_n(\mathbf{p}, \mathbf{x}) \approx \frac{n}{p} f_p^2(\mathbf{p}, \mathbf{x})\,. \tag{6}$$

Averaging over x, f_p could be represented in a much more generalised manner in the following form:

$$\langle f \rangle = \frac{1}{3} \frac{p}{n} \frac{E_d \frac{\mathrm{d}^3 N_d}{\mathrm{d}^3 P}}{E_p \frac{\mathrm{d}^3 N_p}{\mathrm{d}^3 p}}\,. \tag{7a}$$

However, on the basis of the assumption of the non-identical nature of the neutron and proton yields, and with the proper reckoning of the isospin asymmetry between these two species, Wang [7] introduced a correction factor to be represented by $(1+p/n)/2$ as a multiplier, where p/n depends on the combination of the projectile and target species, the collision centrality and nuclear rapidity. Besides, the value of p/n, according to Wang [7], may vary little even with nucleon transverse momentum because of the isospin symmetry of the strong interaction. Thus, replacing in eq. (7a) the factor p/n by this new correction factor, $(1+p/n)/2$, the final working expression takes the following form:

$$\langle f \rangle = \frac{1}{3} \frac{E_d \frac{\mathrm{d}^3 N_d}{\mathrm{d}^3 P}}{E_p \frac{\mathrm{d}^3 N_p}{\mathrm{d}^3 p}} \frac{(1+\frac{p}{n})}{2}. \tag{7b}$$

4 The new approach and the outlook

The generalised form of the inclusive cross-section for production of either proton or deuteron is assumed to be represented here by

$$E\frac{\mathrm{d}^3\sigma}{\mathrm{d}^3 p}|_{PP\to Q+X} = C_1 \left(1 + \frac{p_{\mathrm{T}}}{p_0}\right)^{-n}, \tag{8}$$

where Q stands for proton or deuteron, p_{T} is the transverse momentum of Q, and C_1, p_0, n are the constants. The above form is adaptation of Hagedorn's model [8] for particle production in nucleon-nucleon collisions.

But we are interested in the present study on a particular aspect of the nucleus(A)-nucleus(B) collisions. So, we try to build up a linkage between nucleon-nucleon (PP) and nucleus-nucleus (AB) collisions. With a view to obtaining such a bridge, let us propose here a form as was prescribed first by Peitzmann [9] and was utilised by us before:

$$E\frac{\mathrm{d}^3\sigma}{\mathrm{d}^3 p}|_{AB\to Q+X} \sim (A\cdot B)^{f(p_{\mathrm{T}})} E\frac{\mathrm{d}^3\sigma}{\mathrm{d}^3 p}|_{PP\to Q+X} \tag{9}$$

with the following subsidiary set of relations:

$$f(p_{\mathrm{T}}) = (1 + \alpha p_{\mathrm{T}} + \beta p_{\mathrm{T}}^2)\,, \tag{10}$$

$$E\frac{\mathrm{d}^3 N}{\mathrm{d}^3 p} = \frac{1}{\sigma} E\frac{\mathrm{d}^3 \sigma}{\mathrm{d}^3 p}\,. \tag{11}$$

Using all the expressions from eq. (8) to eq. (11), one obtains, finally,

$$E\frac{\mathrm{d}^3 N}{\mathrm{d}^3 p}|_{AB\to Q+X} = C_2 (A\cdot B)^{(1+\alpha p_{\mathrm{T}} + \beta p_{\mathrm{T}}^2)} \left(1 + \frac{p_{\mathrm{T}}}{p_0}\right)^{-n}, \tag{12}$$

where C_2 is the normalisation constant for the specific $AB \to Q + X$ process. It is to be noted that the factor unity in the exponent of expression (12) could also be lowered, if necessary, with suitable changes only in the normalizing factor of the inclusive cross-section term in the above expression. By our ascription of the form $f(p_{\mathrm{T}})$ given in eq. (10) we introduce first what is called here De-Bhattacharyya parametrisation (DBP). The choice of this form is not altogether a coincidence. In dealing with the EMC effect in the lepton-nucleus collisions, one of the authors here (SB) [10] made use of a polynomial form of the A-dependence with the variable x_{F}(Feynman Scaling variable). This gives us a clue to make a similar choice with both the p_{T} and the $y(\eta)$ variable. In recent times, De-Bhattacharyya parametrisation is being extensively applied to interpret the measured data on the various aspects [11] of the particle-nucleus and nucleus-nucleus interactions at high energies. In the recent past Hwa *et al.* [12] also made use of this sort of relationship in a somewhat different context. The underlying physics implications of this parametrisation stem mainly from the expression (12) which could be identified as a clear mechanism for the switch-over of the results obtained for nucleon-nucleon (PP) collision to those for nucleus-nucleus interactions at high energies in a direct and straightforward manner. The polynomial exponent of the product term on AB takes care of the totality of the nuclear effects. In the next section we would try to give some hints to the physical interpretations of the two parameters α and β used in expression (12).

Dividing eq. (12) for deuteron production in AB collision by the square of that for proton production in the same collision, one would obtain the expression for B_2 with $P_{\mathrm{T}} = 2p_{\mathrm{T}}$, where $P_{\mathrm{T}}(p_{\mathrm{T}})$ denotes the transverse momentum of deuteron (proton). Once B_2 is obtained, the effective source radius R_G for deuteron production in nucleus-nucleus collision can be calculated by eq. (2). In the same way the average phase space density of the proton can be obtained with the aid of eq. (7b) and eq. (12).

5 Hints to physical interpretations of α and β: A preliminary attempt

Indeed, quite obviously, in eq. (10) and eq. (12) above there is a factor unity and there are two phenomenological parameters in $f(p_{\mathrm{T}})$ which need to be physically explained and/or identified. In compliance with this condition we attempt to offer the following modest physical explanations, of only the very suggestive nature, for the occurrence of all these factors. Obviously, such explanations are, at the present stage, only in embryonic form. So, they need thorough and detailed reworking with all the traces of a serious research activity. In fact, we ourselves hope to take up and launch this project quite separately in the near future. Still, we try to offer here what we consider to be the nutshell of the matter. The term unity signifies theoretically the probability of fullest possible participation of either or both the projectile and the target which marks the very onset scenario of any real physical collision. The particle-nucleus or nucleus-nucleus collisions at high energies subsequently give rise to an expanding blob or fireball with rising temperature. In real and concrete terms this stage indicates the growing participation of the already-expanded nuclear blob. As temperature increases at this stage, the emission of highly energetic secondaries (which are mostly peripheral nucleons or baryons) with increasing transverse momentum is perfectly possible. The coefficient α addresses this particularity of the natural event; and this is manifested in the enhancement of the nuclear contribution with the increase of the transverse momentum. Thereafter, there is a turnabout in the state of reality. After the initial fractions of seconds, the earlier-excited nuclear matter starts to cool down and there is a clear natural contraction at this stage, as the system suffers a gradual fall in temperature. Finally, this leads to what one might call “freeze-out” stage, which results in extensive hadronisation, especially in the production of hadrons with very low transverse momentum. In other words, the production of large-p_{T} particles at this stage is lowered to a considerable extent. This fact is represented by the damping or attenuation term for the production of high-p_{T} particles. The factor β with negative values takes care of this state of physical reality. Thus the function, denoted by $f(p_{\mathrm{T}})$, symbolises the totality of the features of the expansion-contraction dynamical scenario in the after-collision stage. Though this interpretation is, at present, only tentative, we make next some simple projections about the quantitative nature of α and β.

In the physics of high-energy nuclear collisions there are some very basic physical facts which determine both the nature of multiplicity of the production of particles and also their inclusive cross-sections. The factors that deserve our attention are the following: a) the total number of parton-parton or binary collisions [13]; b) penetrability within the nuclei to be determined by the role of the kick of the momentum transfer (related with the hardness factor of the collisions) and also by the nature of the impact parameter [14]; c) nature of the parton distribution within the nucleon/nucleus at the pre-excited and excited stage; d) role of rescattering and cascading of the partons through the effects of multiple collisions at high energies; e) the physics of the Glauber model involving basically the concept of impulse approximation and the geometry of the collisions which are described, in the main, by the role of the impact parameter; f) the possible chance of

Table 1. Parameter values for proton and deuteron production in PP collision.

Produced particle	Collision energy	C_1 (mb)	p_0 (GeV/c)	n
Proton	70 GeV/c	18 ± 2	27 ± 3	138 ± 7
Proton	200 GeV/c	$(2.2 \pm 0.8) \times 10^4$	29 ± 5	140 ± 6
Deuteron	70 GeV/c	0.16 ± 0.02	8 ± 1	50 ± 4

structural rearrangement in the nucleon or nucleus by the highly energetic colliding partons.

Summing up all this, the physicists in the realm of heavy-ion physics, suggest and rely, in the main, on two physical parameters for explaining the observations made: i) the total number of binary collisions involving either parton-parton or nucleon-nucleon ones; ii) the number of participating nucleons in a nucleus, denoted by N_{part}. There are, so far, three reported experimental observations of controversial nature: i) the anomalous nuclear enhancement [15,16]; ii) the physics of some sort of EMC effect; iii) the very recent report on the so-called or real suppression of the large-p_{T} particle production at RHIC [17]. Whatever are measured and detected by the experimentalists are to be explained only with the help of the aforestated few conceptual apparatus; and the very basic tools to be employed are $\langle N_{\text{binary}} \rangle$ and $\langle N_{\text{part}} \rangle$. The factors α and β are to be related somehow in terms of these two variables of collisions at high energies.

Thus, intuitively speaking, let us propose that the magnitude of the parameter α in expression (10), is a measure of the ratio of the net binary collision number to the total permissible number among the constituent partons in the pre-"freeze-out" expanding stage identified to be a sort of explosive "detonation" [18] stage. This is approximated by a superposition of collective flow and thermalized internal motion, which is a function of hadronic temperature manifested in the behaviour of the average transverse momentum. The post freeze-out hadron production scenario is taken care of by the soft interaction which is proportional [19,20] to the number of participant nucleons, N_{part}, according to almost any variety of wounded nucleon model. The factor β, we conjecture, offers a sort of ratio of the actual participating nucleons to the total number of maximum allowable (participating) nucleons, denoted by $N_{\max}$. In our future work we would try to substantiate such statements by a careful analysis of the vast array of data for at least 20 collision systems at various energies.

6 Model-based analyses and results

The values of the factors p_0 and n in expression (8) for the production of proton in PP collision at 70 GeV/c and at 200 GeV/c, and those for the production of deuteron in PP collision at 70 GeV/c are given in table 1. It would have been extremely logical to do the same for the deuteron production exactly at 200 GeV/c. But due to lack of reliable PP data for deuteron production at 200 GeV/c, we have assumed here that the values of p_0 and n

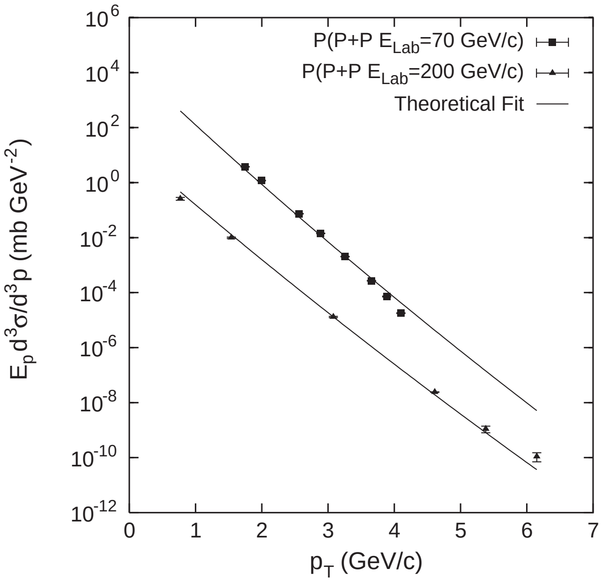

Fig. 1. Plot of $E\frac{\mathrm{d}^3\sigma}{\mathrm{d}^3p}$ *vs.* p_{T} for protons produced in PP collision at 70 GeV/c and at 200 GeV/c. The filled squares and triangles represent the experimental data points [21,16]. The solid curves provide the theoretical fits on the basis of eq. (8).

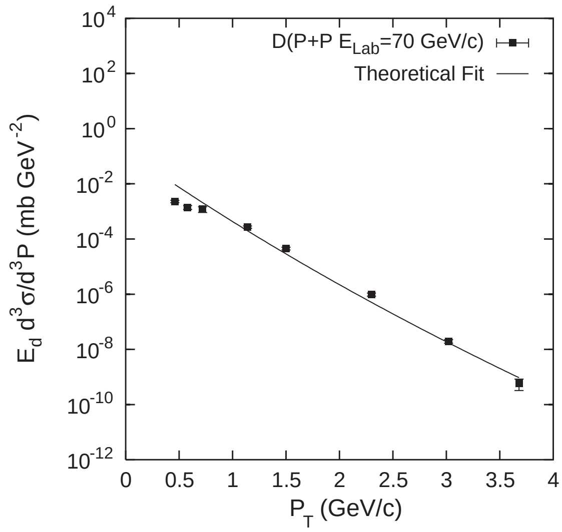

Fig. 2. The inclusive spectra for the production of deuterons in PP collision at 70 GeV/c as a function of transverse momentum, p_{T}. The filled squares represent the experimental data points [22]. The solid curve depicts the theoretical fit on the basis of eq. (8).

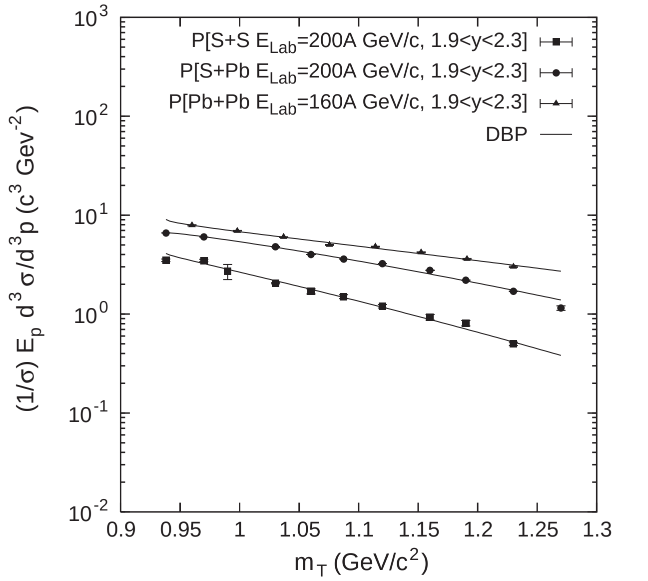

Fig. 3. The nature of $E\frac{\mathrm{d}^3N}{\mathrm{d}^3p}$ as a function of proton transverse mass for production of protons in SS, SPb and PbPb collisions. The experimental data points are taken from ref. [6]. The solid curves represent the DBP-based fits obtained on the basis of eq. (12).

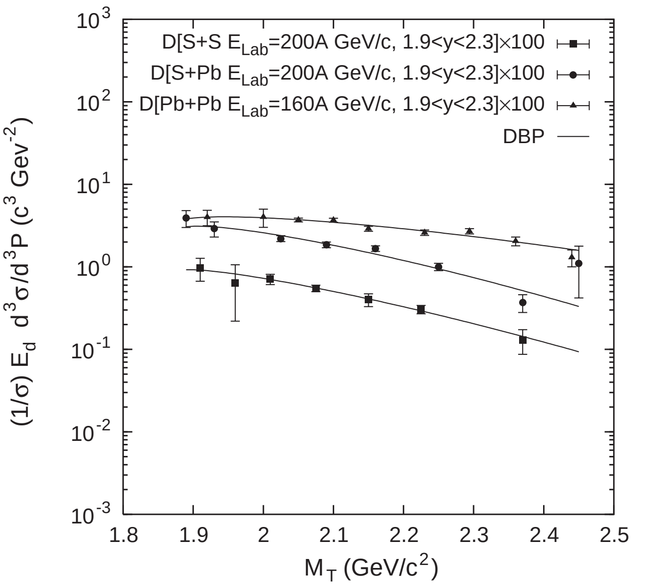

Fig. 4. The invariant spectra for production of deuterons in SS, SPb and PbPb collisions. The experimental data-points are taken from ref. [6]. The solid curves represent the DBP-based fits obtained on the basis of eq. (12).

for the deuteron production at 200 GeV/c will remain the same as those for the case at 70 GeV/c. The justification for this assumption will be clearer if one carefully examines the first two rows of table 1, wherein the parameter values of p_0 and n for proton production in PP collisions at 70 GeV/c and at 200 GeV/c show up very close and too neighbouring values even in quantitative terms.

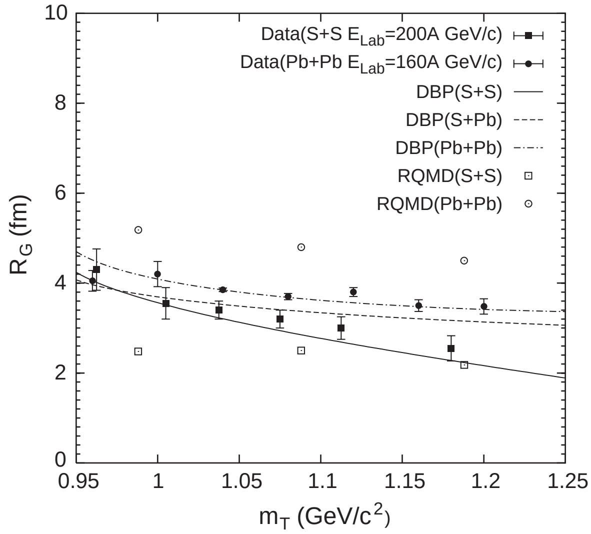

Fig. 5. Plot of the effective source radius R_G as a function of proton transverse mass in SS, SPb and PbPb collisions. The filled symbols represent the experimental data while the open symbols are for RQMD results [6]. The curves are on the basis of the present approach (DBP). For the case of SPb collision we have simply drawn the DBP-based curve as a sort of prediction, as no data or RQMD predictions on those collisions are still available. The symbols and the curves present also a comparison between the performance by the RQMD and the present approach.

The figures in the diagrams (figs. 1 and 2) represent the behaviours of eq. (8) against the measured data [16, 21] with the fitted parameters given in table 1. It is seen that the expression offers a good description of the data on the nature of p_{T} spectra for the production of protons (fig. 3) and deuterons (fig. 4) in a few selected nuclear collisions which we have presented for only three collisions involving SS, SPb and PbPb. The data presented by Murray and Holzer [6] prescribes nearly 10% centrality for all three collisions. With minor adjustments of only these two parameters α and β and of the normalization factor, we could describe data on the transverse momentum (or rapidity spectra) of the various secondaries in different centrality regions and in diverse rapidity bins. This statement is based on one of our very recent studies [11].

Our next task is to assign values of C_2, α and β for the parameters in expression (12) which help us to achieve satisfactory reproduction of the experimental values of the source radius and the APSD values for SS and PbPb collisions shown in the diagrams presented in figs. 5, 6 and 7. The values of C_2, α and β which finally lead us to the results on source radii and APSD values for both proton and deuteron production in different collisions are given separately in table 2 and table 3, respectively.

7 Final discussion and conclusions

The graphical representations of the results based on the calculations by De-Bhattacharyya parametrization

Table 2. Different parameter values for proton production in different nucleus-nucleus collisions.

Collision	C_2	α (c/GeV)	β (c^2/GeV^2)
SS (200 A GeV/c)	0.004 ± 0.001	0.64 ± 0.03	$-(0.44 \pm 0.04)$
SPb (200 A GeV/c)	0.0010 ± 0.0003	0.56 ± 0.04	$-(0.28 \pm 0.03)$
PbPb (160 A GeV/c)	0.00020 ± 0.00005	0.40 ± 0.04	$-(0.11 \pm 0.02)$

Table 3. Different parameter values for deuteron production in different nucleus-nucleus collisions.

Collision	C_2	α (c/GeV)	β (c^2/GeV^2)
SS (200 A GeV/c)	$(8.2 \pm 0.5) \times 10^{-6}$	1.0 ± 0.3	$-(0.24 \pm 0.02)$
SPb (200 A GeV/c)	$(4.0 \pm 0.3) \times 10^{-6}$	0.82 ± 0.06	$-(0.20 \pm 0.02)$
PbPb (160 A GeV/c)	$(7.7 \pm 0.6) \times 10^{-7}$	0.66 ± 0.04	$-(0.11 \pm 0.01)$

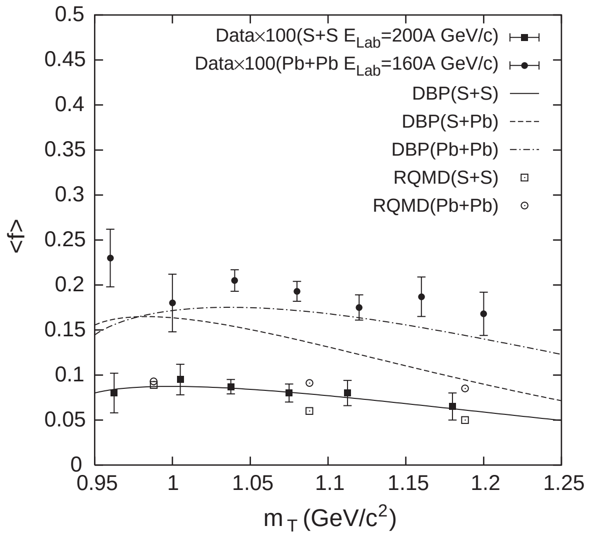

Fig. 6. The nature of average phase space density $\langle f \rangle$ as a function of proton transverse mass in SS, SPb and Pb + Pb collisions. The filled symbols represent the experimental data while the open symbols are for RQMD results [6]. The curves are drawn on the basis of the DBP-based calculations. For the case of SPb collision we have simply drawn the DBP-based curve as a sort of prediction, as no data or RQMD predictions on the same collisions are still available. A comparison of performances by the RQMD and the present approach is thus essentially obtained here.

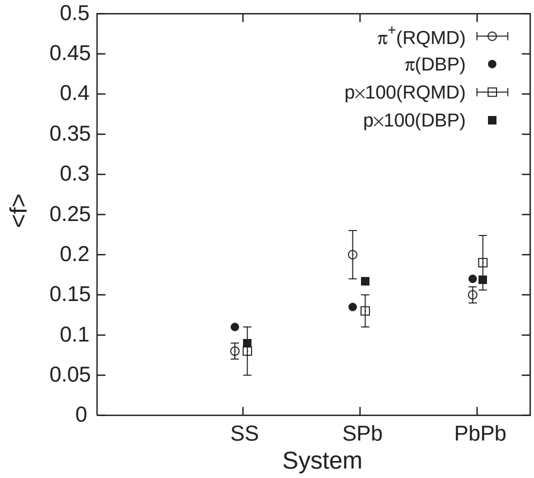

Fig. 7. Comparison of pionic and protonic average phase space densities($\langle f \rangle$) at $\langle p_{\mathrm{T}} \rangle \approx 240$ MeV/c in different nucleus-nucleus collisions between the DBP-based results and the RQMD calculations. The source of RQMD results is in ref. [6], while the DBP-based results for pions are from refs. [1,2].

(DBP) are clearly shown in all the diagrams of figs. 3, 4. The nature of the agreement is quite satisfactory. The factors plotted along the ordinate in these graphs are the measured observables. Speaking in relative terms, however, the calculated results for the invariant cross-section for deuteron production do not show agreement as is reflected in the proton cases for various $AB \sim AA$ collisions. The equivalent radii of the effective expansion sphere (presented in fig. 5) are satisfactorily obtained by the calculations based on a combination of the phenomenological models and the new parametrization proposed here. Indeed, the agreements obtained for the cases of SS and PbPb collisions are quite striking. The issue of SPb collision, which apparently constitutes an exception here, will be taken up for short discussion in the next paragraph. It would be obvious from an analysis of fig. 5 that the agreements obtained by the DBP-based calculations are somewhat better than those by RQMD. The observations are the same or similar to the average phase space density (APSD) values ($\langle f \rangle$) of protons shown in fig. 6. So, the new parametrisation tried and tested here for analysing the APSD values of protons produced in nuclear collisions succeeds satisfactorily.

Let us emphasise the next point separately. Figure 7 demonstrates the nature of $\langle f_p \rangle$ and $\langle f_\pi \rangle$ with the interacting systems. But drawing any definitive conclusion on this dependence is somewhat difficult at this stage, as the measurements on the invariant cross-section for the case of SPb collisions suffer from a considerable degree of uncertainty; and this peculiarity of SPb collisions is in contrast with the data on the other two collisions studied here. These points become clear and evident, as soon as one looks at the nature of the fits provided in fig. 4. Even in the calculations based on the RQMD model, these fits are reflected by the large uncertainty ranges. However, it could safely be stated here that both the sets of values of APSD (for both pion and proton) show, on the average, a modest increase with the effective mass number of the systems. Secondly, the APSD values of protons are, quite consistently, far less than those for pions; and this reduction (for the case of protons) is roughly by two orders of magnitude. In terms of the physics of the so-called expansion dynamics, this boils down to the statement that the freeze-out for the production of protons occurs i) much later and ii) much more rarely than for the production of pions. This is only natural for any valid physical scenario.

Next, some concrete comments are in order here right now. We do make no explicit claim here to be a competitor of the Relativistic Quantum Molecular Dynamics (RQMD) which constitutes a semi-classical microscopic approach with a combination of classical propagation, stochastic interactions and the tenets of the string theory ideas and the hadronic resonances. Despite having the semblance of the constituent approaches of RQMD rooted in the first principles in the relevant sectors/field, nuclear light cluster formation is not included dynamically in RQMD for which an afterburner is normally applied for deuteron yields calculations. This inherent crack in the RQMD mechanism brings it qualitatively somewhat nearer to our own approach comprising two phenomenological models and a new parametrisation for the production of particle spectra. For the case of deuteron-antideuteron studies, this is specially true in spite of its high degree of theoretical flavour, its technical sophistication, wide acceptability, and the numerous applications to the studies of relativistic heavy-ion collisions at high energies. Secondly, that the phenomenological approach with two free parameters adopted by us here gives nice and, to some extent, better fits to the data, at times, may not appear to be too exciting at first sight, and may appear to be just a coincidence. But, one must take note of the simple fact that the model was not initially introduced for studying light cluster formation; it was formulated first to analyse the very basic nature of the rapidity and transverse momentum spectra for production of pions, kaon-antikaons, baryon-antibaryons etc. And this has now been extended to obtain the features of deuteron production in heavy-ion collisions with fair success. This serendipity might, thus, reasonably heighten our optimism about the present compact approach to heavy-ion collisions.

Quite noticeably, we have studied here the nuclear interactions at the same energy. So, the c.m. energy dependence of the APSD character and of the radii of an hypothetical sphere are left out of the purview of the present work. Besides, the studies on the production of a baryonic anti-particle, that is, on $\bar{p}$ and on an antiparticle cluster like $\bar{D}$, have been reported in separate communications.

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